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TRANSIENT ANALYSIS OF GAS TURBINE POWER PLANTS IN DIFFERENT CASES OF DETERIORATIONS

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Abstract

Most monitoring systems of gas turbine engines based on steady state operation and diagnostic analysis have traditionally been conducted under steady state conditions. Analysis shows that the transient operational processes have significant diagnostic content. This paper describes the mathematical models of transient operational modes of gas turbine engines, presents results of transient analysis by these models at different deteriorations. Finally the diagnostic opportunities are discussed.

Keywords: mathematical model, gas turbine, compressor, diagnostics, turbojet, turbofan.

1. Principle of Building Mathematical Models

The mathematical models are based on those of the component parts, which – as a matter of fact – represent their computing programs of characteristics.

The following characteristics:

inlet duct $\pi^* = f(M),$ (1)

compressor $\pi_c^* = f[q(\lambda)_c, n_c],\quad \eta_c = f[q(\lambda)_c, n_c],$ (2)

turbine $q(\lambda)_t = f(\pi_t^*, \lambda_u),\quad \eta_t = f(\pi_t^*, \lambda_u),$ (3)

nozzle $q(\lambda_n) = f(\pi_n)$ (4)

can be given at the form of polynomials reflecting the results of the preliminary calculations or the measurements, or else they can be inserted into the skeleton program as subroutines, respectively.

In expressions (1)–(4) π^*, π — stagnation and static pressure ratio, $q(\lambda)$ — dimensionless mass-flow rate, n_c — corrected rotational speed, η — efficiency, λ_u — dimensionless velocity, M — flight Mach number. Subscripts c, t, n are compressor, turbine, nozzle, respectively.

The component parts are connected by the skeleton-program which, at the same time, ensures the fulfilment of the law of conservation.

The set of skeleton equations forming the mathematical model is constituted of the conservation equations of the possible mass and energy stores found in the power plant, as well as of the rules of controlling (n = const., T_3 = const., etc).

There are three kinds of energy stores to be distinguished: the mass store, the mechanical energy store and the heat store. The mass stores represent the interspace between the components (compressors, turbines, nozzle) taking part in the conversion process of heat-mechanical energy. Mechanical energy is stored by the rotors, while heat energy is stored by the metallic parts and gases.

The general equation of mass stores accordingly will be

$$\dot{m}(j) - \dot{m}(j+1) = \frac{dm}{dt}, \tag{5}$$

$$m = \frac{pV}{RT}, \tag{6}$$

where T and p can be given by equations

$$\frac{dT}{dV} = \frac{\left\{\left(\dot{m}(j+1)T(j+1) - \dot{m}(j)T(j)\right)\frac{c_p}{c_v} - T\left(\dot{m}(j+1) - \dot{m}(j)\right)\right\} RT}{pV},$$
$$\frac{dp}{dt} = \frac{RT}{V}\left(\dot{m}(j) - \dot{m}(j+1)\right) + \frac{p}{T}\frac{dT}{dt} \tag{7}$$

and $\dot{m}(j), \dot{m}(j+1)$ – mass-flow rates entering and leaving the store, m, V, T, p – mass, volume, mean temperature and pressure of gas in store, c_p, c_v – isobaric and isochoric specific heats, $T(j)$, $T(j+1)$ – store inlet and outlet temperatures, respectively, t – time.

The storage of the mechanical energy in the rotors will take place with the help of the following equation:

$$P_t(i) - P_c(i) - \Delta P(i) = 4\pi^2 n(i)\Theta(i)\frac{dn(i)}{dt}, \tag{8}$$

where P_t, P_c — actual turbine and compressor power, ΔP — power extracted from i-th rotor, n — number of revolutions, Θ — polar moment of inertia.

The thermal energy to be stored in the metallic parts and gases can be taken into consideration by the following relationship:

$$m_m c_m \frac{dT_m}{dt} + V c_p \frac{d(\rho_g T_g)}{dt} = \dot{m}(j)c_pT(j) - \dot{m}(j+1)c_pT(j+1), \tag{9}$$

where m_m, c_m, T_m — mass, specific heat, temperature of metal parts, ρ_g, T_g — density, and temperature of gas.

The time delay occurring in the processes can be calculated for the individual cases on the basis of the following equation:

$$\tau = \frac{pV}{RT\dot{m}} \tag{10}$$

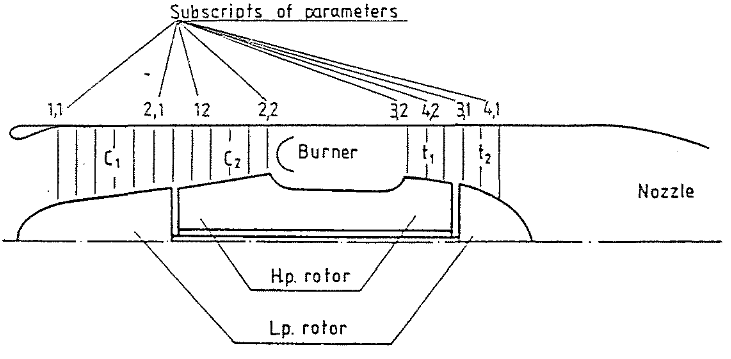

Fig. 1.

In the case of the twin-spool jet-engine shown in *Fig. 1*, the set of equations will have the following form:

$$\dot{m}_c(2) - \dot{m}_c(1) - \Delta\dot{m}(1) = \frac{dm_{c12}}{dt}, \tag{11}$$

$$\dot{m}_c(2) - \Delta\dot{m}(2) + \dot{m}_f - \dot{m}_t(2) = \frac{dm_b}{dt}, \tag{12}$$

$$\dot{m}_t(2) + \Delta\dot{m}_r(2) + \Delta\dot{m}_s(1) - \dot{m}_t(1) = \frac{dm_{t12}}{dt}, \tag{13}$$

$$\dot{m}_t(1) - \Delta\dot{m}_r(2) - \dot{m}_n = \frac{dm_{tn}}{dt}, \tag{14}$$

$$P_t(i) - P_c(i) - \Delta P(i) = 4\pi^2\Theta(i)n(i)\frac{dn(i)}{dt}, \tag{15,16}$$

$$(i = 1, 2)$$

rule of controlling. (17)

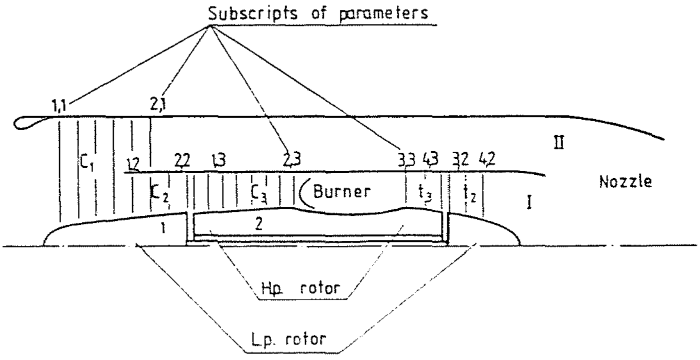

Fig. 2.

Here, accordingly, $\dot{m}_c, \dot{m}_t, \dot{m}_n$ — inlet compressor, turbine and nozzle mass-flow rates, $\Delta\dot{m}$ — air flow extracted from the compressor, $\Delta\dot{m}_r, \Delta\dot{m}_s$ — masses of air recirculated into the gas flow for cooling the rotor and stator blades, $\dot{m}_f$ — fuel flow rate, m_{c12}, m_{t12} — masses of gases in volumes between compressors and turbines, m_b, m_{mix} — masses of gases in burner and mixing chamber, m_{tn} — mass of gases in volumes between low-pressure turbine and nozzle.

For twin-spool turbofans equipped with primary fan-stages and a constant-area mixer (*Fig. 2*), the system is formed from the following equations:

$$\frac{\dot{m}_c(1)}{\alpha+1} - \dot{m}_c(2) = \frac{dm_{c12}}{dt}, \tag{18}$$

$$\dot{m}_c(3) - \dot{m}_c(2) - \Delta\dot{m}(2) = \frac{dm_{c23}}{dt}, \tag{19}$$

$$\dot{m}_c(3) - \Delta\dot{m}(3) + \dot{m}_f - \dot{m}_t(3) = \frac{dm_b}{dt}, \tag{20}$$

$$\dot{m}_t(3) + \Delta\dot{m}_r(3) + \Delta\dot{m}_s(2) - \dot{m}_t(2) = \frac{dm_{t23}}{dt}, \tag{21}$$

$$\dot{m}_c(1)\frac{\alpha}{\alpha+1} - \dot{m}_2 = \frac{dm_{m2}}{dt}, \tag{22}$$

$$\dot{m}_t(2) + \Delta\dot{m}_r(2) - \dot{m}_1 = \frac{dm_{m1}}{dt}, \tag{23}$$

$$\dot{m}_1 + \dot{m}_2 - \dot{m}_n = \frac{dm_{mix}}{dt}, \tag{24}$$

$$P_t(2) - P_c(1) - P_c(2) - \Delta P(2) = 4\pi^2 \Theta(1) n(2) \frac{dn(2)}{dt}, \tag{25}$$

$$P_t(3) - P_c(3) - \Delta P(3) = 4\pi^2 \Theta(2) n(3) \frac{dn(3)}{dt}, \tag{26}$$

$$p_I - p_{II} = 0, \tag{27}$$

$$\text{rule of controlling} \tag{28}$$

is obtained, where α – the bypass ratio, $\dot{m}_1, \dot{m}_2$ – mass-flow rates of primary and secondary streams, m_{m1}, m_{m2} – masses of gases in stores before mixing chamber, m_{mix} — mass of gases in mixing chamber, p_I, p_{II} – static pressures of streams at the entrance into mixing chamber.

Eq. (27) serves as the condition of solution to the conservation equations of energy, momentum and mass-flow rates as written for the mixer. Within the set of non-linear, skeleton differential equation formed in this way, each of the equations constitutes a closed unit in itself. According to the flow path direction of gases, the equations can be completed with the use of the relationships provided by gas turbine theory.

In the models, the gas characteristics and the specific heat were determined as temperature- and composition-dependent ones in each case. In the equations, it was suitable to calculate with the normed, relative values — i.e. values related to the nominal duty — of the unknowns. In this way, the roots are yielded as of nearly identical order of magnitude, which — in turn — will take an influence on the convergence of the method, too, through the accuracy. The required partial derivatives were replaced by difference quotients.

The examinations carried out with the help of the full multi-store set of equations clearly show that in the case of the modelled power plants, it is sufficient to take into consideration the energy-storing effect of the rotors, only. The influence of the other stores, as well as that of the non-steady-state heat transfer between the metallic parts and gases is practically negligible, or it can be compensated in the course of model adaptation. The same holds for the deviations due to the assumption of the originally quasi-steady-state process, and for the modifications in the characteristics occurring with transient processes.

Possible rules controlling the transient process will be

$$\dot{m}_f = f(t)$$

or with coastdown it may have the form:

$$\dot{m}_f = 0.$$

The set of non-linear differential equations obtained in this way was solved with the help of the fourth-order Runge–Kutta method.

The modelling equation of the steady-state operational modes can be obtained with the zero values of the derivatives with respect to time. According to the rule of controlling in case of turbojet engine $n(1) = \text{const.}$, but in case of turbofan $n(3) = \text{const.}$ To solve the set of non-linear equations,the Newton–Raphson method proved to be the most effective [1], [2].

2. Adaptation of Mathematical Models to the Objects Modelled

To describe the modelled gas turbine equipment in the best possible way with the models built up in the way introduced above, it is required to carry out the adaptation of the model on the basis of the measurement results.

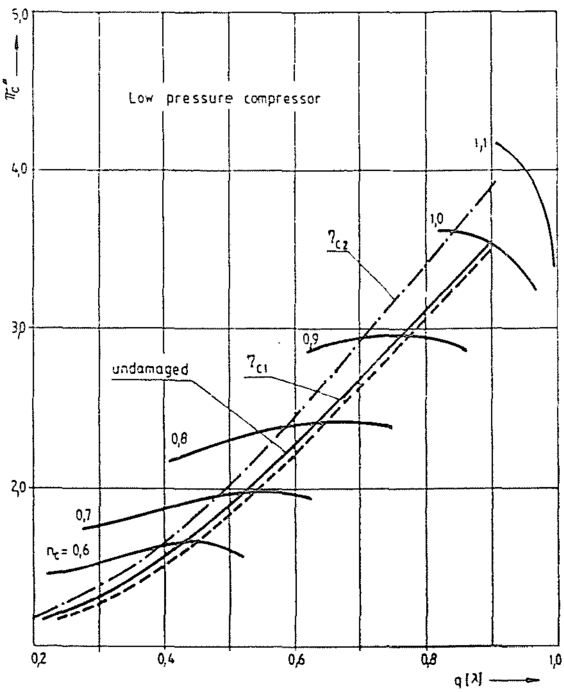

Fig. 3.

This means that the different internal constants of the mathematical model are changed in a way that its set of equations should remain unchanged, however, the solutions provided by it should draw nearest to the measurement results [1].

The adaptation of transient processes, the different operational duties will represent the power-plant states at the different points of time. For performing the adaptations, processed (filtered and smoothed) results should be used.

The aim of adaptation can be the correction of the compressor and turbine characteristics.

3. Application of Models to Analyzing the Behaviour of Gas Turbine Power Plants

The adapted model is suitable for the examination of the processes taking place in the gas-turbine power plants, for the determination of such parameter values which will not undergo measurement. The analysis of the behaviour of power plants can be performed in the case of different simulated conditions, damage. In *Figs. 3,4*, the modification of the operating line of a twin-spool turbojet as occurred due to the reduction of compressor efficiencies can be seen on the maps of the low- and high-pressure compressors, respectively. In the Figures it is visualized clearly that the decreasing of the high-pressure compressor efficiency shifts the operating line towards the surge line on the maps of the low- and high-pressure compressors.

4. Application of Models in Diagnostics

With the help of the elaborated models, examinations of parameter sensitivity can be performed both in steady-state and transient operational duties. These examinations show that with the individual state characteristics (efficiencies, cross-sectional areas, losses, etc.) the sensitivity of the power-plant parameters measured or measurable potentially will be different. On the basis of such examinations, the diagnostic matrix of the given power plant can be generated [3].

Investigations shows [1], [2], that the sensitivity of the individual parameters varies in a different measure on the different operational duties, as well as the ratio of those variations relative to each other. In Fig. 5, the change in the relative parameter sensitivity (T_{41} and n_1) is shown occurring with the transient (accelerating) and steady duties of the turbojet (*Fig. 1*) in case of low-pressure turbine efficiency reduction illustrated versus high-pressure rotor rotational speed. *Fig. 6* shows the effect of fan efficiency reduction on relative sensitivity of turbofan (*Fig. 2*) in transient (accelerating) and steady duties.

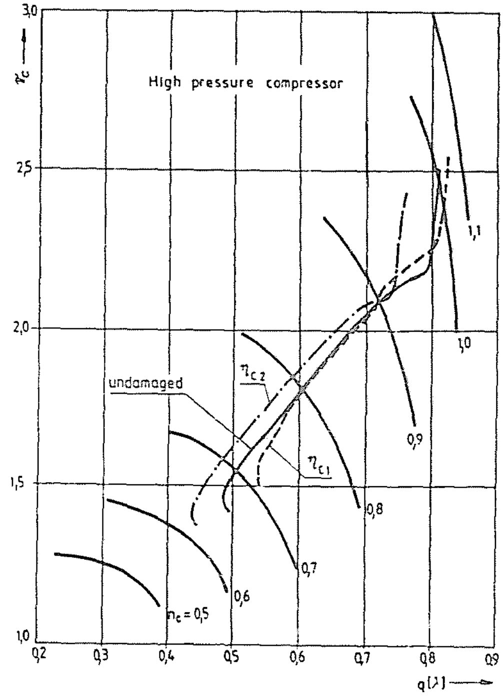

Fig. 4.

In *Fig. 7* the change in relative sensitivity is shown in case of deceleration of turbojet.

Comparison of diagrams in *Fig. 5–Fig. 7* shows that accelerating process bends the sensitivity curves upward positive direction, but in case of decelerating process opposite effect is found. According to the results of examinations, if the relative sensitivity of the individual parameters is positive then its magnitude is generally greater with the transient processes than with the steady-state operational modes. In decelerating processes the negative sensitivity parameters increase (by absolute value).

Consequently, a controlled transient process can provide additional information about the state of the power plant and has diagnostic content. Inasmuch as the already adapted, undamaged (etalon) model is adapted to the processed measurement results of the damaged power plant with the help of the corresponding adaptation program [1], [2], then the cause of damage can be traced back more accurately.

Naturally, every power plant should have its own adapted model. The adaptation is carried out by the program in a way that the characteristic

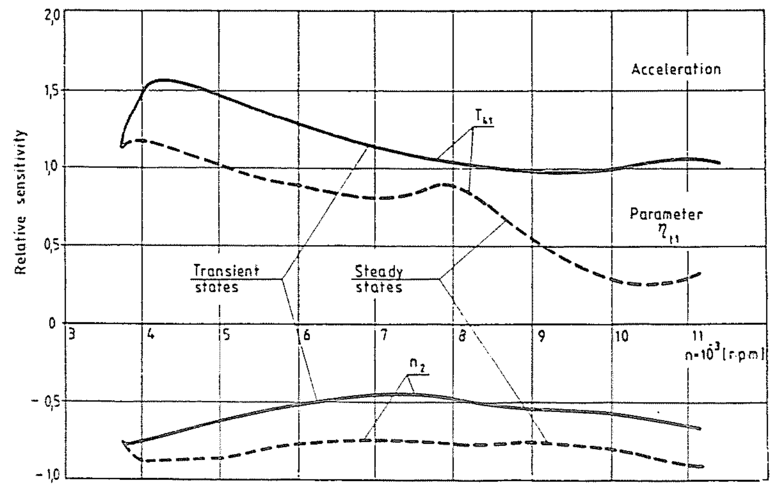

Fig. 5.

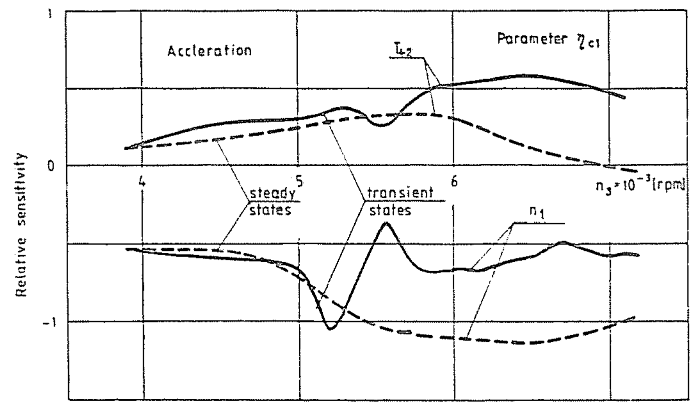

Fig. 6.

efficiencies, losses, etc. of the power plant will be determined under identical conditions (nominal duty, standard environmental circumstances) on the basis of measurements performed on a wide range of operational duties. In this way, the variation of the given characteristic can be evaluated, the trend of variation can be followed after appropriate (e.g. exponential) smoothing.

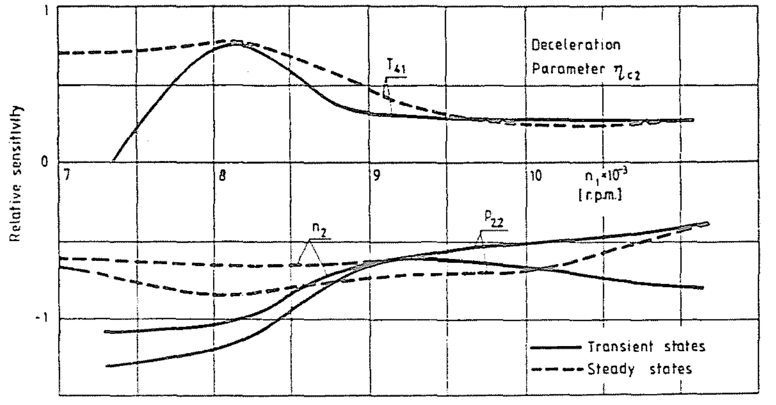

Fig. 7.

5. Summary

This paper discusses a possibility of modelling of transient and steady operational modes of gas turbine power plants, provides some results of their transient analysis. The results of investigations show valuable diagnostic content of transient processes.

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